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Original Article

SECOND-ORDER SLIDING-MODE CONTROL FOR A PRESSURIZED WATER NUCLEAR REACTOR CONSIDERING THE XENON CONCENTRATION FEEDBACK

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A B S T R A C T

This paper presents findings on the second-order sliding-mode controller for a nuclear research reactor. Sliding-mode controllers for nuclear reactors have been used for some time, but higher-order sliding-mode controllers have the added advantage of reduced chattering. The nonlinear model of Pakistan Research Reactor-1 has been used for higher-order sliding-mode controller design and performance evaluation. The reactor core is simulated based on point kinetics equations and one delayed neutron groups. The model assumes feedback from lumped fuel and coolant temperatures. The effect of xenon concentration is also considered. The employed method is easy to implement in practical applications, and the second-order sliding-mode control exhibits the desired dynamic properties during the entire output-tracking process. Simulation results are presented to demonstrate the effectiveness of the proposed controller in terms of performance, robustness, and stability.

1. Introduction

An accurate mathematical model of a system is essential for the correct development of control systems. Designs based on incorrect plant models will result in poorly performing plant controllers. Nuclear reactors are highly complex, nonlinear, time-varying, and constrained systems, and therefore, accurate model development and validation are very difficult. Plant parameters vary with time. For example, the nuclear plant characteristics vary with operating power levels, aging effects, and changes in nuclear core reactivity with fuel burnup. These parameter variations make accurate model development a difficult task [1].

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Sliding-mode control has gained much importance in control applications and has been widely applied. Because of its simple structure, inherent robustness, and capability to control nonlinear systems, it has gained attention as a subject of research, and a variety of new algorithms have been proposed. Second-order sliding-mode control has recently been proposed [2–4] and is being applied in new applications. The main advantage of this algorithm is that higher-order derivatives of the states are not required and a chattering-free control was derived. Another advantage of sliding-mode control is that it can be easily implemented due to its simple structure.

The second-order sliding-mode control algorithm has been applied for regulating the output power of Pakistan Research Reactor 1 (PARR-1). PARR-1 is a 10-MW swimming pool-type research reactor. It was provided by American Machine Foundry and was first made critical on December 21, 1965. The instrumentation and control system was upgraded in 1986 and the reactor power was upgraded from 5 MW to 10 MW in 1992. The high-enriched uranium fuel was replaced with light-enriched uranium fuel in the same year (Table 1).

A second-order sliding-mode controller has been developed around the nonlinear simulation model. There are five control rods. The reactor becomes critical when these rods are withdrawn by about 56% from their fully inserted position. After the reactor becomes critical, a single rod is used to regulate the reactor power [5].

A sliding-mode controller has been applied in nuclear reactor control applications. Huang et al. [6] have used fuzzy adaptive recursive sliding-mode controllers for controlling reactor pressure, reactor water level, and turbine power. The chattering problem was minimized using the recursive sliding-mode algorithm. Shtessel [7] applied a sliding-mode controller to space nuclear reactor control systems and showed good improvements in simulated results. Robustness and good tracking were reported. Huang and Edwards [8] applied a recursive sliding-mode controller to control advanced boiling water reactor turbine throttle pressure by controlling turbine control valve opening. It was proven by simulation results that the recursive sliding-mode control algorithm results in milder control actions and considerably weaker power surges than the conventional proportional-integral controller. In this study, a higher-order sliding-mode control system (HOSMC) was designed for core power control of the research nuclear reactor to improve the load following capability. The method used for controller synthesis is the second-order sliding mode based on the super twisting algorithm.

Table 1 – Parameters values of PARR-1.

Symbol	Quantity	Symbol	Quantity
β	0.007275	μ_f	0.333
Δ	0.000041	μ_c	0.1451
f_f	0.95	λ	0.079147
α_f	1.11×10^{-6}	T_{f0}	30
α_c	-5.97×10^{-5}	T_{c0}	30
P_0	0.001	M	5.7351×10^{-5}
Ω	0.81577		

PARR-1, Pakistan Research Reactor 1.

2. Nuclear reactor core model

To simulate the nuclear reactor core, point kinetics equations with one group of the delayed neutrons are used [9]. The model assumes feedback from lumped fuel and coolant temperatures. The effect of xenon concentration is also included. The normalized model, with respect to an equilibrium condition, based on point kinetics equations with three delayed neutron groups are as follows:

$$\frac{dn_r}{dt} = \frac{\rho(t) - \beta}{\Lambda} n_r(t) + \frac{\beta}{\Lambda} c_r \tag{1}$$

$$\frac{dc_r}{dt} = \lambda n_r(t) - \lambda c_r(t) \tag{2}$$

$$\frac{dT_f}{dt} = \frac{1}{\mu_f}\left[f_f P_0 n_r(t) - \Omega(T_f - T_c)\right] \tag{3}$$

$$\frac{dT_l}{dt} = \frac{1}{\mu_c}\left[\left(1 - f_f\right) P_0 n_r(t) + \Omega(T_f - T_c) - M(T_l - T_e)\right] \tag{4}$$

$$\frac{dx}{dt} = \gamma_x \Sigma_f \phi + \lambda_I I - \sigma_x x \phi - \lambda_x x \tag{5}$$

$$\frac{dI}{dt} = \gamma_I \Sigma_f \phi - \lambda_I I \tag{6}$$

$$\rho = \rho_r + \alpha_f(T_f - T_{f0}) + \alpha_c(T_c - T_{c0}) - \left(\frac{\sigma_x}{\Sigma_f}\right)(x - x_0) \tag{7}$$

$$\frac{d\rho_r}{dt} = G_r Z_r \tag{8}$$

where

$n_r \frac{n}{n_0}$, Neutron density relative to initial equilibrium density;.

n, Neutron density(n/cm^3);

n_0, Initial equilibrium (steady − −state) neutron density;.

$c_{ri} \frac{c_i}{c_{i0}}$, relative density of ith group precursor;.

c_i, Core averaged ith group precursor density$\left(\frac{atom}{cm^3}\right)$;.

c_{i0}, Initial equilibrium (steady − −state) density of ith group precursor. ;

ρ, $\frac{(k-1)}{k}$, reactivity.;

k_{eff}, Effective neutron multiplication factor;

k k_{eff}, effective neutron multiplication factor.

Λ Effective prompt neutron life time (seconds).;

λ_i, Radioactive decay constant of the ith group neutron precursor (seconds^{-1});.

β, Total delayed neutron fraction;.

β_i, ith gGroup delayed neutron fraction.

T_f, Average reactor fuel temperature (°C).;

T_l, Temperature of the water leaving the reactor (°C).;

T_e, Temperature of the water entering the reactor (°C);.

$T_c(T_1+T_e)/2$, aAverage reactor coolant temperature (°C);

f_f, Fraction of reactor power deposited in the fuel;.

P_0, Initial equilibrium power (MW);.

μ_f, Total heat capacity of the fuel $=$ weight of fuel times its specific heat $\left(\frac{MW.s}{°C}\right)$;.

μ_c, Total heat capacity of the reactor coolant $=$ weight of coolant times its specific heat $\left(\frac{MW.s}{°C}\right)$;

Ω, Heat transfer coefficient between fuel and coolant (MW/°C);

M, Mass flow rate multiplied by heat capacity of the coolant (MW/°C);.

$\delta\rho_r$, Reactivity due to control rod movement;.

Z_r, Control input, control rod speed in units of fraction of core length per second;.

G_r, Total reactivity worth of control rod;.

α_f, Fuel temperature reactivity coefficient $\left(\frac{\Delta k}{k}\Big/°C\right)$.;

α_c, Coolant temperature reactivity coefficient $\left(\frac{\Delta k}{k}\Big/°C\right)$.;

T_{f0}, Initial equilibrium (steady – state) fuel temperature.;

T_{c0}, Initial equilibrium (steady – state) coolant average temperature.;

σ_x, Microscopic absorption cross – –section of xenon (cm^2).;

λ Xenon decay constant (1/second).;

λ_I, Iodine decay constant $\left(\frac{1}{second}\right)$.;

$\gamma - x$, Xenon yield.;

γ_I, Iodine yield.;

and

ϕ, Neutron flux $\left(\frac{n}{cm^2 \cdot s}\right)$

Equations 3 and 4 represent change in fuel and coolant temperatures. Equations 5 and 6 are related to the changes in xenon and iodine concentrations. Equations 7 and 8 represent changes in reactivity due to temperature and xenon feedback and control rod position change.

3. Reactor control problem

The basic operation of a reactor relies on the fission process. As the mass of ^{235}U becomes greater than the critical mass (supercritical), the fission process starts automatically. The fission is controlled by inserting control rods, which absorb neutrons, in parallel with fuel rods. As the control rods are moved up, the fission process starts automatically when the control rods achieve a particular height. After criticality, only a single rod is moved up or down to control the reactor power. Usually light water is used as a moderator, which also helps in controlling the fission rate. Water is also used as a coolant to maintain the reactor core temperature under the desired level. Four general methods are possible for changing the neutron flux in a reactor; they involve temporary addition or removal of fuel, moderator, reflector, or a neutron absorber (poison). Each of these methods or a combination of them has been used for reactor control, but the procedure most commonly used is the insertion or withdrawal of a material, such as boron or cadmium, having a large cross section for the absorption of neutrons [10]. The absorber and the fissile material may be regarded as competitors for neutrons; the larger the proportion absorbed by the control material, the smaller the fraction available for fissions, and vice versa. The control elements are referred to as control rods because they were originally, and often still are, long cylindrical rods (or a combination of such rods). For regulating the output power of PARR-1, the control rod motion is used as a control variable.

4. Second-order sliding-mode controller design

Higher-order sliding-mode controllers have started to find useful applications in different control problems. Qaiser et al.

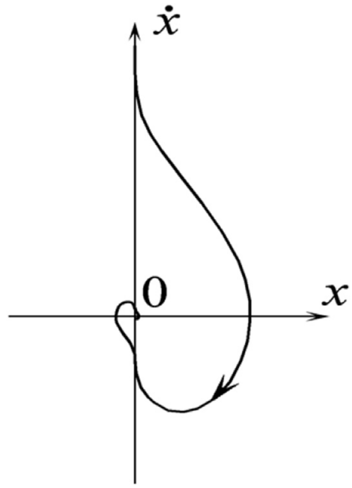

Fig. 1 – Super twisting controller: phase portrait.

[1] have applied the higher-order sliding-mode technique in nuclear reactors, but without considering the effect of xenon concentration. In this paper, a second-order sliding-mode control system is presented for core power control of the research nuclear reactor to improve the load following capability. The effect of xenon concentration is also included in a reactivity feedback model.

The main difference between the conventional and higher-order sliding modes is that higher-order derivatives of the sliding variable s are used in place of first-order derivatives [3,4,11]. In addition to the advantages of first-order sliding modes, HOSMC has the additional advantage of chattering removal. The n^{th} order sliding mode is determined by the equalities $s = \dot{s} = \ddot{s} = ... = s^{n-1}$, which form the n-dimensional condition on dynamic system state [12]. Control law $u(t)$ is defined as a combination of two terms in a super twisting algorithm. The first one is defined as a discontinuous time derivative whereas the second term is the continuous function of the sliding variable. The trajectories of the super twisting algorithm are characterized by twisting around the origin on the sliding variable phase portrait. The corresponding phase portrait is shown in Fig. 1 [13].

Consider the system of the following form:

$$\dot{y}_1 = y_2 \tag{9}$$

$$\dot{y}_2 = \theta(t,x) + \gamma(t,x)u \tag{10}$$

where $y_1 = s$, $y_2 = \dot{s}$, and $\theta(t,x)$, $\gamma(t,x)$ are smooth uncertain functions with $|\theta| \le \Theta > 0, 0 < \gamma \le \Gamma_M$.The super twisting algorithm converges to the two-sliding set ($s = \dot{s} = 0$) in finite time and can be given as:

$$u(t) = u_1(t) + u_2(t) \tag{11}$$

$$\dot{u}_1 = \begin{cases} -u & |u| > 1 \\ -W sign(s) & |u| \le 1 \end{cases} \tag{12}$$

$$u_2 = \begin{cases} -m|s_0|^{\rho} sign(s) & |s| > s_0 \\ -m|s|^{\rho} sign(s) & |s| \le s_0 \end{cases} \tag{13}$$

where $W > 0,\ 0 < \rho \le 1,\ and\ 0 < s_0 > |s(t,x)|$. The sliding surface has been taken as $= n_r - n_{rd}$, the error between the desired and existing neutron flux or reactor power. It satisfies the second-order differential equation of the form:

$$\bar{s} = \theta(x) + \gamma(x)u \tag{14}$$

where $\theta(x) = \tilde{\theta}(x) - \ddot{x}_{1d}$.

For the range of interest:

$$|\theta| \le \Theta > 0;$$

$$0 < \Gamma_m \le \gamma \le \Gamma_M$$

The control law can be defined as:

$$u(t) = -m|s|^{\rho} sign(s) + u_1, \tag{15}$$

The value of m determines the overshoot and steady-state error. By increasing the value of m, overshoot decreases, but at the same time steady-state error increases. Conversely, by making the value small, perfect tracking is achieved at the cost of higher overshoot. The factor w controls the speed of convergence. For small values of w, the output converges slowly to its final value, whereas for the bigger value it converges promptly. There is no need for any information about time derivatives of the sliding variable, s, nor about system parameters. This reduces the need for an observer, and therefore, computational complexity is reduced. For finite-

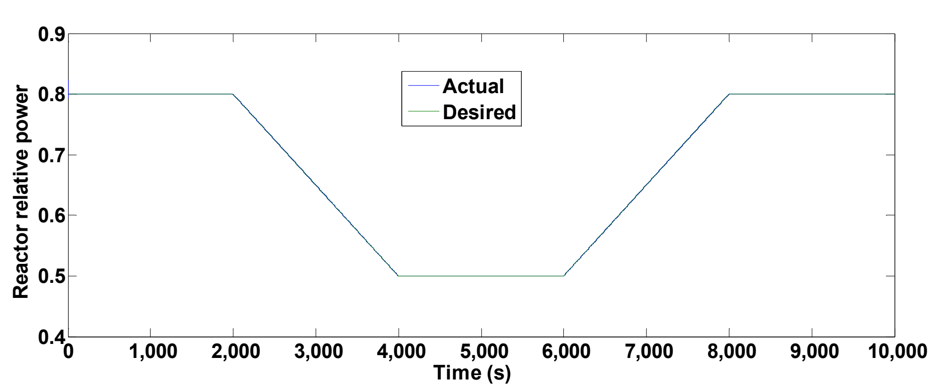

Fig. 2 – Reactor relative power (80–50–80%).

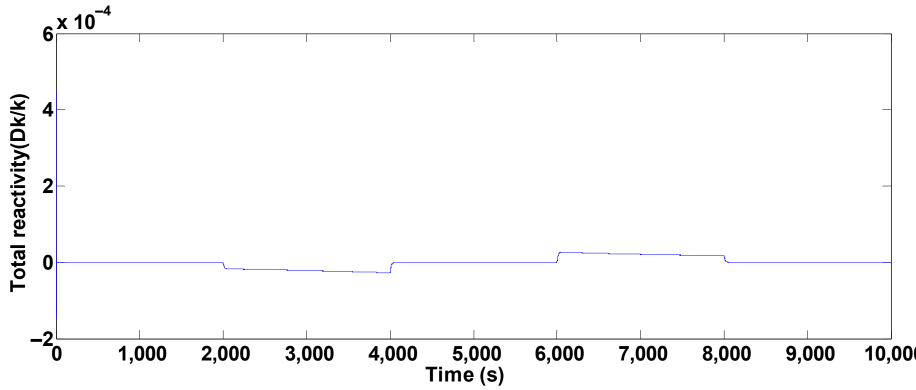

Fig. 3 – Total reactivity of the core.

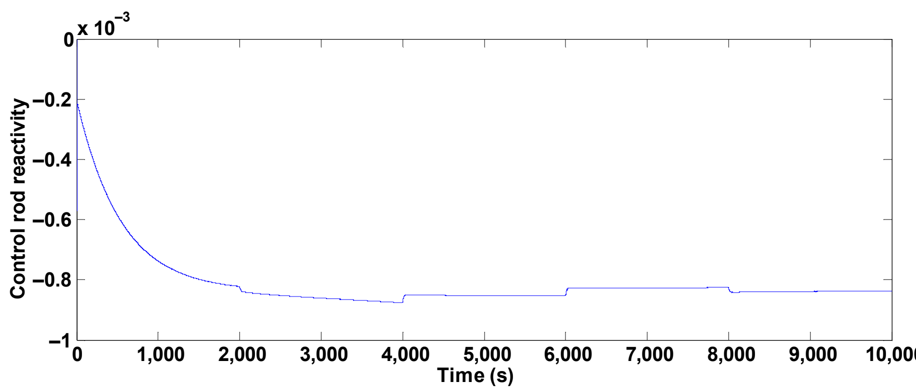

Fig. 4 – Control rod reactivity.

time convergence proof of the super twisting algorithm, the reader is referred to [14].

5. Simulation results

In this section, a set of simulations performed on the reactor model described in the "Nuclear reactor core model" section is evaluated to understand the performance and robustness of the proposed control structure. Two different transients have been used to evaluate the performance of the controller. In all of these cases, the objective is to follow the reference power in the reactor. The sliding surface is taken as the difference between the desired and the actual neutron flux level. The input taken is the change in reactivity, and the output is the neutron flux level n_r. Fig. 2 shows the output tracking of the neutron flux to a normalized flux demand (80–50–80%) along with controller effort. The desired power is reached quickly with no overshoot or oscillation and with a minimal control effort (Figs. 3, 4).

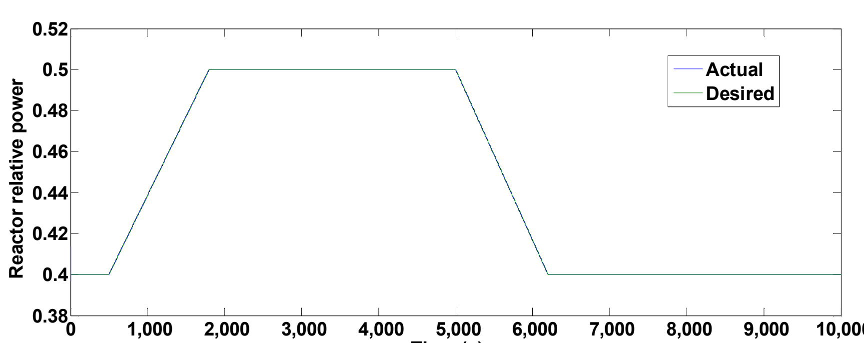

Fig. 5 – Reactor relative power (40–50–40%).

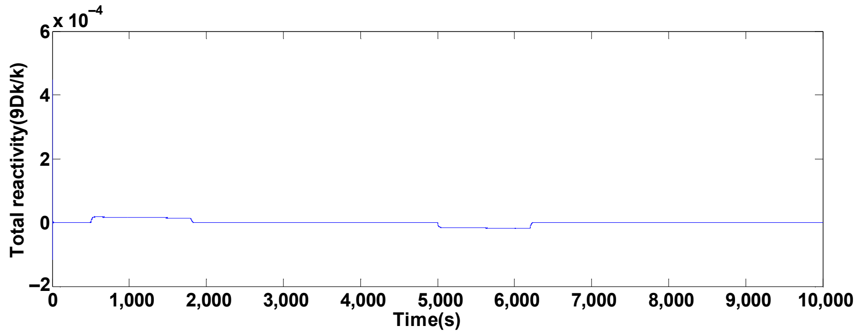

Fig. 6 – Total reactivity of the core.

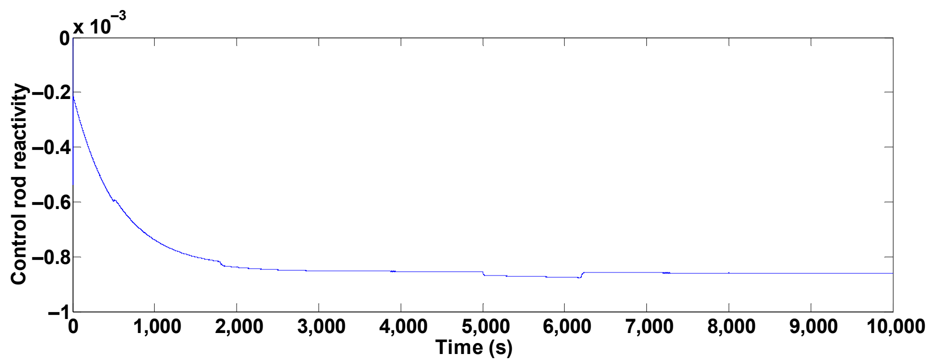

Fig. 7 – Control rod reactivity.

Fig. 5 shows the output tracking of the neutron flux to a normalized flux demand (40–50–40%) along with controller effort. Such as in the previous case, the desired power is reached quickly with no overshoot or oscillation and with a minimal control effort (Figs. 6, 7).

The disturbance rejection capability of the designed controller is shown in Figs. 8, 11. The control rod reactivity with disturbance insertion (80–50–80%) is shown in Fig. 9. The disturbance is a pulse-type disturbance with 20% magnitude that is shown in Fig. 10. As seen in Figs. 8, 11, the disturbance is rejected almost immediately again with the minimal controller effort. Fig. 12 shows the control rod reactivity with disturbance insertion (40–50–40%). The output remains stable, thus proving the robustness of the controller. Sensitivity analysis, by varying the parameter values, was also verified. The controller still behaved well in dealing with parameter uncertainties.

6. Conclusion

In this paper, a second-order sliding-mode control system has been presented for core power control of the pressurized water reactor nuclear reactor to improve the load following capability. The reactor core was simulated based on the point kinetics equations and one delayed neutron groups. The model assumed feedback from lumped fuel and coolant temperatures. The effect of xenon concentration was also

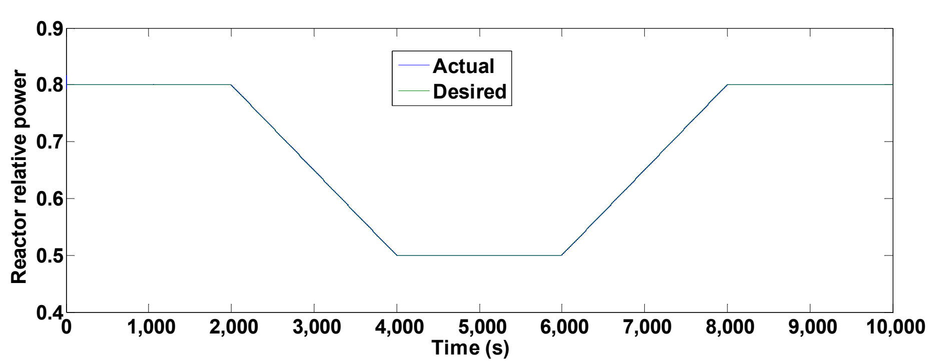

Fig. 8 – Disturbance rejection of reactor relative power (80–50–80%).

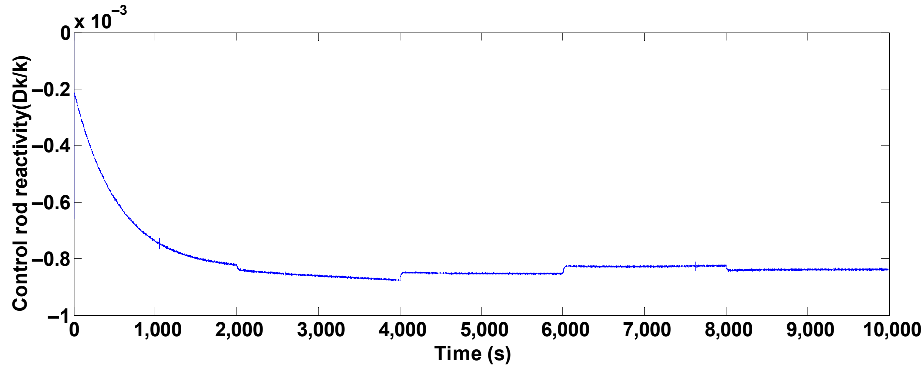

Fig. 9 – Control rod reactivity with disturbance insertion (80–50–80%).

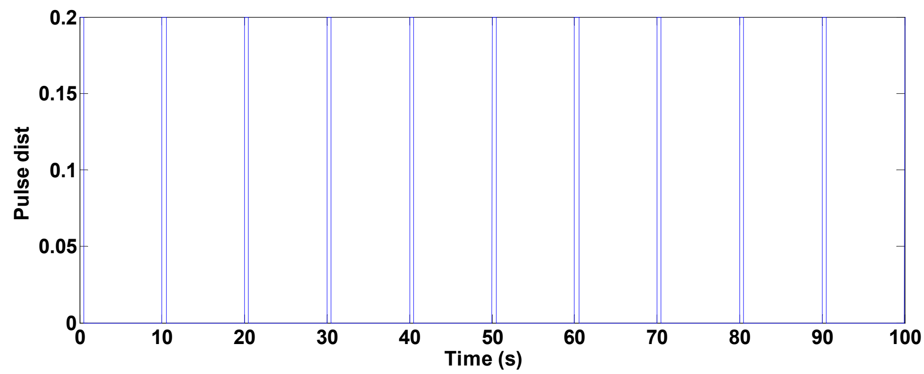

Fig. 10 – Pulse-type disturbance.

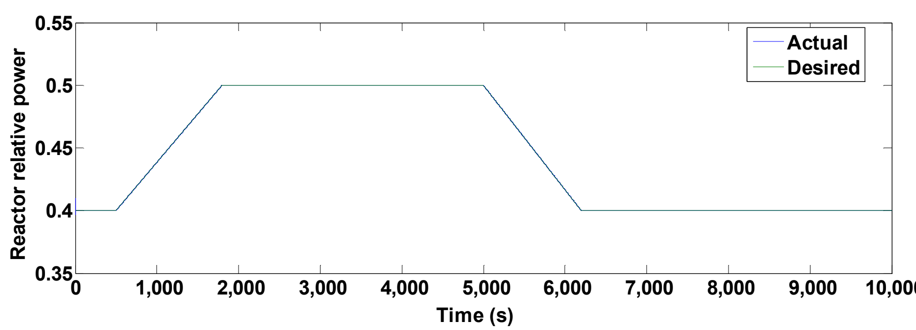

Fig. 11 – Disturbance rejection of reactor relative power (40–50–40%).

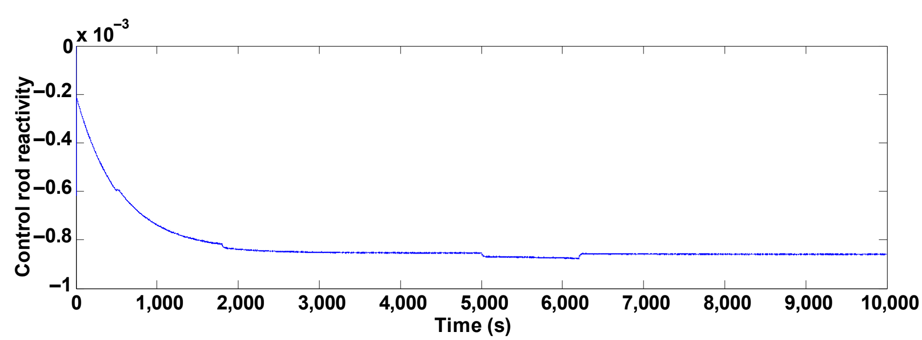

Fig. 12 – Control rod reactivity with disturbance insertion (40–50–40%).

included in the model. The control system was guaranteed to be stable within a large range. The main advantage of the HOSMC technique is its inherent insensitivity and robustness to plant uncertainties and external disturbances. It is also without the controller chattering, which is common in first-order sliding modes. This approach provides a high-performance controller on the system. Moreover, the simplicity of the controller structure and design procedure, as well as the satisfactory tracking performance and robust stability, shows notable improvements compared with the previous designs.

Conflicts of interest

The authors declare no conflicts of interest.

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